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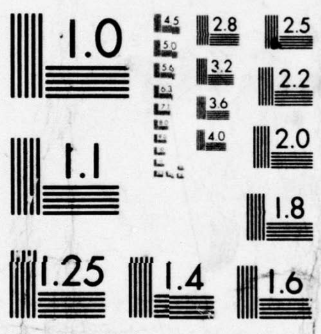

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MEMORANDUM REPORT NO. 2700

THE RESPONSE OF LINEAR UNSTABLE SYSTEMS TO STATIONARY GAUSSIAN EXCITATION

Pierre Lafrance

November 1976

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USA BALLISTIC RESEARCH LABORATORIES
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When an unstable, linear mechanical system is perturbed by random ambient disturbances, the dynamical parameters (velocities, pressure) characterizing the system are themselves characterized by statistical and spectral distributions that can be directly evaluated from a statistical and spectral description of the random initial condition. This is a natural extension of the deterministic approach whereby a given initial condition (say, a sinusoidal perturbation) gives rise to a deterministic outcome (say, a time-increasing next page

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20. ABSTRACT (continued)

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sinusoidal amplitude). While the deterministic approach is valid only when the initial conditions are known exactly, it must be superseded by a stochastic model when these conditions are known only in a statistical sense. This model is successfully applied to the breakup of liquid jets where turbulence is assumed to provide the random disturbance.

TABLE OF CONTENTS

I. INTRODUCTION

In classical stability analysis, one observes the behavior of small departures from the equilibrium state. These disturbances decay in stable systems, retain their initial amplitude in neutrally stable systems, and are amplified in the unstable case. It is often convenient to choose these small perturbations from a set of orthogonal and complete functions since in the linear case any arbitrary initial perturbation can be expressed as a linear combination of members from this set so that one may, without loss of generality, confine one's analysis to an initial perturbation which is described by a single but arbitrary member of this set. In many cases, the set par excellence consists of sinusoidal functions. In addition to their obvious mathematical convenience, they are easily reproduced experimentally so that a direct comparison between theory and experiment is possible. In these experiments, the initial perturbation is usually too small to be directly perceptible. As it grows in time, it manifests itself macroscopically, the effect of the instability being similar to an amplifier for this small disturbance. Such controlled experiments are characterized by a high degree of regularity which disappears as soon as the disturbance-producing mechanism is removed. Here, the unstable system obtains its initial perturbation from the random ambient vibrations always present in its environment. Consequently, the macroscopic behavior of this system is also random. A deterministic description of this process is ruled out since the initial perturbation is not known exactly. However, a knowledge of the statistical and spectral properties of the initial perturbation is sufficient to determine the statistical and spectral properties of the unstable system after the initial perturbation has been applied.

This paper discusses the response of an unstable linear system to stationary gaussian noise with zero mean. The treatment is very similar to that of calculating the output of a linear filter with stochastic input but with one major difference: the present analysis involves both space and time variables. The random functions describing system parameters are stationary and gaussian in the space coordinates, but generally increasing functions of time (because of the instability). We show that when a certain parameter (say, a flow velocity) is perturbed with stationary gaussian noise with a given spectral density, the perturbation grows in time but remains gaussian - its standard deviation being a function of time. Furthermore, when the spectral density becomes confined to a narrow band of wave numbers, the amplitude of the perturbation is Rayleigh distributed. The present model is found to be in good agreement with the data of Phinney[1], suggesting a simple model for the breakup of turbulent liquid jets; one may view the turbulence as a source of random

1. *R. E. Phinney, "The Breakup of a Turbulent Liquid Jet in a Gaseous Atmosphere," J. Fluid Mech., Vol. 60, 1973, p. 689.*

perturbations for the liquid jet. Since the turbulence amplitude is small, one could expect the linear breakup theory of Rayleigh[2] to be applicable, with modifications described in the present report to take into account the stochastic nature of the perturbation.

II. LINEAR THEORY

In linear stability theory, one can often decompose perturbations of the field variables (e.g., velocity components, pressure, movable boundary coordinates) into elementary components of the form

$$\exp i(kx - \omega t)$$

where x and t are space and time coordinates respectively, $k = 2\pi/\lambda$ is the wave number of that component (with wavelength λ) of the perturbation applied at time $t = 0$, and $\omega(k) = \omega_1(k) + i\omega_2(k)$ is the complex frequency. We can express a typical field variable, denoted by $\psi(x,t)$ by a linear superposition of these elementary components:

$$\psi(x,t) = \int_k \Psi(k)\, e^{i(kx-\omega_1 t)}\, e^{\omega_2 t}\, dk \qquad (1)$$

where $\Psi(k)$ is the Fourier transform of the initial perturbation. Eq. (1) can be written in the form

$$\psi(x,t) = \int_k F(k,t)\, \Psi(k)\, e^{ikx}\, dk \qquad (2)$$

where

$$F(k,t) = e^{-i\omega_1(k)t}\, e^{\omega_2(k)t} \qquad (3)$$

This form is reminiscent of a linear filter with input waveform $\psi(x,o)$ at $t = 0$ and output waveform $\psi(x,t)$ at time t. $F(k,t)$ is the time-evolving Fourier transform of the impulse response.

When the input (initial condition) is stochastic, then the output $\psi(x,t)$ will also fluctuate randomly. For the sake of definiteness, and mathematical tractability, we consider the initial conditions to have the following properties in the stated coordinate:

a. Stationary

b. Gaussian with zero mean

2. *Lord Rayleigh, Theory of Sound, Vol. II, Dover, New York (1945), pp. 351-355.*

c. Ergodic

The system is assumed to be linear. Time plays the role of a family parameter which we consider fixed for any input - output linear operation. Since the system is linear, a gaussian input will result in a gaussian output. We characterize our random process by its autocorrelation function. This characterization represents a simple statistical average based on the second order probability distribution function for the process. The autocorrelation function is defined by

$$R(y,t) = \lim_{z\to\infty} \frac{1}{2z} \int_{-z}^{z} \psi(x,t)\ \psi(x+y,t)\,dx \tag{4}$$

The spectral density for this process is obtained from the Wiener-Khintchine relation

$$G(k,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(y,t)\ e^{-iky}\ dy \tag{5}$$

whence

$$R(y,t) = \int_{-\infty}^{\infty} G(k,t)\ e^{iky}\ dk \tag{6}$$

We now write the input - output relation for the spectral densities

$$G(k,t) = |F(k,t)|^2\ G(k,o) \tag{7}$$

where the input is applied as an initial condition with spectral density $G(k,0)$ at time $t = 0$ and the output is expressed in terms of either its autocorrelation function $R(y,t)$ or of its spectral density $G(k,t)$. Since $G(k,0)$ is given, we can view $|F(k,t)|^2$ as a time evolution operator which relates the state of the system at time t to the given initial conditions. We can easily find an explicit form for the time evolution operator. Using Eqs. (3) and (7), we have

$$G(k,t) = e^{2\omega_2(k)t}\ G(k,0) \tag{8}$$

so that the spectral evolution depends only on the imaginary part of the complex frequency.

Consider now the set of stationary random gaussian field variables with zero mean

$$\{\psi_n\} = \{\psi(x_{n,t})\} \tag{9}$$

3. *W. B. Davenport, Jr., and W. L. Root, An Introduction to the Theory of Random Signals and Noise, McGraw Hill, New York (1958).*

The probability of measuring ψ_1 at $x = x_1$, ψ_2 at $x = x_2$, . . . , ψ_N at $x = x_N$, all at time t is[3]

$$p\{\psi_N\}\, d\psi_1 \ldots d\psi_N = \frac{d\psi_1 \ldots d\psi_N}{(2\pi)^{N/2} |\Lambda|^{\frac{1}{2}}} \exp \frac{-1}{2|\Lambda|} \left[\sum_{n,m=1}^{N} |\Lambda|_{m,n} \psi_m \psi_n \right] \tag{10}$$

where Λ is the co-variance matrix with elements

$$\lambda_{m,n} = R(x_m - x_n, t) \tag{11}$$

and co-factors $|\Lambda|_{m,n}$. For a stationary process, let $y = x_m - x_n$; also define $\sigma^2(t) = \sigma^2 = R(0,t)$. We then have

$$\Lambda = \begin{vmatrix} \sigma^2(t) & R(y,t) & \ldots & R(y,t) \\ R(y,t) & \sigma^2(t) & \ldots & R(y,t) \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ R(y,t) & R(y,t) & & \sigma^2(t) \end{vmatrix} \tag{12}$$

with

$$R(y,t) = \int_{-\infty}^{\infty} G(k,o)\, e^{2\omega_2(k)t}\, e^{iky}\, dk \tag{13}$$

and

$$\sigma^2(t) = \int_{-\infty}^{\infty} G(k,o)\, e^{2\omega_2(k)t}\, dk \tag{14}$$

We have taken advantage of the fact that for a stationary process the multivariate gaussian distribution with zero mean is totally specified by the auto-correlation function, which in turn is related to the initial condition by Eq. (13).

III. WIDEBAND INPUT WITH DOMINANT MODE

We proceed by studying a particularly simple but yet often encountered form for $G(k,0)$ and $\omega_2(k)$. Imagine the initial condition to be gaussian white noise

$$G(k,0) = 1 \tag{15}$$

and $\omega_2(k)$ to be strongly peaked at $k = k_o$

$$\omega_2(k) > 0 \tag{16a}$$

$$\frac{d\omega_2}{dk} = 0 \quad \text{at } k = k_o \tag{16b}$$

$$\frac{d^2\omega_2}{dk^2} < 0 \text{ at } k = k_o \tag{16c}$$

Note that Eq. (15) represents an assumption about the environment while Eq. (16) is based upon a description of the physical system. In practice, these quantities can be measured and the exact techniques of the previous section may be applied. In a large number of cases, however, Eqs. (15 - 16) represent a sufficiently good approximation to reality, for example, a liquid jet subjected to wideband vibrations (which are assumed to have Gaussian statistics). We start by expanding $\omega_2(k)$ about k_o and keeping only the first two non-vanishing terms:

$$\omega_2(k) = \omega_2(k_o) + \tfrac{1}{2}\,(k - k_o)^2\,\omega_2''(k_o) \tag{17}$$

since $\omega_2'(k_o) = 0$. Eq. (8) can be used to calculate the spectral density, which takes the form of a Gaussian distribution

$$G(k,t) = A(t)\,\exp\left[-\tfrac{1}{2}\left(\frac{k-k_o}{\Sigma_s}\right)^2\right] \tag{18}$$

with amplitude

$$A(t) = e^{2\omega_2(k_o)t} \tag{19}$$

and standard deviation

$$\Sigma_s = \left[\,2\,|\omega_2''(k_o)|\,t\right]^{-\frac{1}{2}} \tag{20}$$

The auto-correlation function is readily obtained:

$$R(y,t) = \sqrt{2\pi}\ A(t)\ \Sigma_s \cos(k_o y) \exp[-\tfrac{1}{2}(y\Sigma_s)^2] \tag{21}$$

and

$$\sigma^2(t) = e^{2\omega_2(k_o)t} \left[\frac{\pi}{|\omega_2''(k_o)|t}\right]^{\frac{1}{2}} \tag{22}$$

The singularities in Eqs. (20) and (22) need not be alarming; they merely indicate that the spectral and statistical distributions of the perturbation tend toward uniform distributions as t becomes small. Examination of Eq. (22) shows that σ (t) decreases from a very large value at t = 0 to a minimum at $t = 1/4\omega_2(k_o) = \tau$ and thereafter increases without bound. Typically, τ is a very small quantity. For example, in a liquid jet, it is of the order of one-fiftieth of the breakup time. This singular behavior at small times is but a reminder that white noise with infinite bandwidth is unphysical.

Note from Eq. (20) that as time increases the harmonic contents of the perturbation tend to become concentrated in a narrow band of wavenumbers centered about $k = k_o$. Functions such as these, which have a sharply peaked spectral density, can be written in the narrowband form[4,5]

$$\psi(x,t) = \xi(x,t) \cos[k_o x - \omega_1 t + \phi(x,t)] \tag{23}$$

where $\xi(x,t)$ and $\phi(x,t)$ represent the envelope and phase of $\psi(x,t)$ respectively, and are slowly varying functions of x. $\psi(x,t)$ owes its almost sinusoidal appearance to the predominance of spectral components at $k = k_o$. The presence of adjoining components gives rise to slow changes in the envelope and phase. If the phase is uniformly distributed (random phase approximation), then it can be shown[5] that the following statistical distributions hold

$$f_\phi(\phi) = \frac{1}{2\pi} \tag{24a}$$

$$f_\xi(\xi) = \frac{\xi}{\sigma^2} \exp[-\tfrac{1}{2}(\tfrac{\xi}{\sigma})^2] \tag{24b}$$

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4. *J. S. Bendat, Principles and Applications of Random Noise Theory, John Wiley & Sons, New York (1958).*

5. *M. Schwartz, Information Transmission, Modulation and Noise, McGraw Hill, New York (1970).*

$$f_\psi(\psi) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}\left(\frac{\psi}{\sigma}\right)^2\right] \quad . \tag{24c}$$

Although we have shown narrowband signals to arise from the preferential amplification of spectral components near the wave number k_o, it is easy to see that they can also be produced when $G(k,0)$ is sharply peaked at $k = k_o$, say.

IV. APPLICATION TO TURBULENT JET BREAKUP

Phinney[1] has measured the statistical distribution of the breakup length of turbulent liquid jets. The breakup length is defined as the length of the continuous portion of the jet, measured from the nozzle to the breakup point where drop formation occurs. For artificially perturbed laminar jets (e.g., with an imposed sinusoidal vibration of constant amplitude), the breakup length is a constant whose magnitude depends on the amplitude and wavelength of the imposed perturbation as well as on hydrodynamic parameters such as jet diameter, surface tension and fluid density. When no artificial disturbance is used, or when the jet is turbulent, the breakup length fluctuates about an average value. Phinney states that the distribution of the turbulent breakup length is described well by a Gaussian but remarks that there exists no theoretical background in which to fit the observation. We do not propose such a theory here, but have posed the following question. Can we achieve a reasonably good correlation with experimental data by assuming that turbulent jet breakup occurs as a result of random perturbations acting on an otherwise *laminar capillary* jet? We shall assume that Rayleigh's linear theory[2] for the breakup of inviscid capillary jet applies, with the sole modification that the small initial perturbation is allowed to fluctuate randomly. Thus, the role of turbulence is limited to that of a driving mechanism which initially deforms the jet surface in a random manner. This ruffled surface initiates the process of capillary instability which ultimately results in jet breakup. Rayleigh's criterion for breakup can be written as

$$\tfrac{1}{2} = \xi_0 \exp\left[\omega\ (k) \left(\frac{8}{N_w}\right)^{1/2} L\right] \tag{25}$$

where ξ_0 is the initial perturbation amplitude, $N_w = \rho d_0 U_0{}^2/T$ is the Weber number, d_0 is the equilibrium jet diameter, U_0 is the jet velocity, T is the surface tension and ρ is the density of the fluid. Eq. (25) states that the breakup occurs when the initial amplitude ξ_0 (rendered dimensionless with respect to d_0) has been exponentially

amplified to the size of the jet diameter. The breakup time and jet velocity have been combined to give L, the breakup length measured in jet diameters. The function $\omega(k)$ is the amplification growth rate and depends on the wavenumber of the perturbation

$$\omega(k) = \left[\frac{8T}{\rho d_0^3} k (1-k^2) \frac{I_1(k)}{I_0(k)}\right]^{\frac{1}{2}} \tag{26}$$

where k is the dimensionless wavenumber and I_0 and I_1 are modified Bessel functions. Eq. (25) provides a deterministic relationship between the initial perturbation amplitude and jet breakup length. If however the initial perturbation amplitude is not fixed, but is distributed according to some statistical law, say $f_{\xi_0}(\xi_0)$, then the breakup length will be distributed according to $f_L(L)$, the two distribution variables being related by Eq. (25). One can express one statistical distribution in terms of the other by a simple variable transformation which conserves probability

$$f_L(L) = f_{\xi_0}(\xi_0) \left|\frac{d\xi_0}{dL}\right| \tag{27}$$

Our problem then is reduced to that of finding $f_{\xi_0}(\xi_0)$. To this end we note that $\omega(k)$ has a peak at $k_0 = 0.698$. The effect of this is to preferentially filter initial excitations so that only a narrow range of wavenumbers centered about k_0 can significantly participate in the breakup process. It has been shown[4,5] that the statistical distribution of the amplitude in a narrowband process is

$$f_{\xi_0}(\xi_0) = \frac{\xi_0}{\sigma_0^2} \exp\left[-\tfrac{1}{2}\left(\frac{\xi_0}{\sigma_0}\right)^2\right] \tag{28}$$

where σ_0 is the r.m.s. value of the random amplitude. Eqs. (25), (27) and (28) can be used to obtain

$$f_L(L) = \frac{\Omega}{4\sigma_0^2} e^{-2\Omega L} \exp\left[-\frac{1}{8\sigma_0^2} e^{-2\Omega L}\right] \tag{29}$$

where

$$\Omega = \left(\frac{8}{N_w}\right)^{\frac{1}{2}} \omega(k_0) \tag{30}$$

Eq. (29) has a maximum at

$$\langle L\rangle = \frac{1}{2\Omega}\ \ell n\ \left(\frac{1}{8\sigma_0{}^2}\right) \tag{31}$$

which is the mode of that distribution and is used in defining a conveniently normalized breakup length

$$\ell = \frac{L}{\langle L\rangle} \tag{32}$$

Phinney's measured breakup lengths were normalized to the mean breakup length and were plotted[1] to show that these data exhibit gaussian scattering about the mean. For comparison with the present theory, Phinney's data have been replotted along with our theoretical breakup length distribution. The result is shown in Fig. (1), where the only free parameter, σ_0, has been adjusted to minimize the mean square error. σ_0 was thus determined to have the value 0.008, a reasonable figure for the r.m.s. amplitude of turbulent fluctuations; unfortunately, no experimental value was reported. The fit is not uniformly good, but a chi-square analysis revealed that the level of confidence is better than 97% for $0.85 \leqslant \ell \leqslant 1.15$ and drops rapidly for values of ℓ outside this interval. This is consistent with the interpretation that values of ℓ clustered around unity arise from small fluctuations of the initial perturbation amplitude about some mean value. Significantly larger fluctuations cannot be handled by the linear model presented here.

V. CONCLUSION

In the first part of this paper, we have calculated the effect of stationary gaussian noise with zero mean on unstable linear systems. The random noise was considered an initial condition rather than a forcing function. The development paralleled that of the response of a linear filter to random signals. These results were specialized to the important case where the initial disturbance is white noise, and where the instability preferentially amplifies a narrow band of the Fourier spectrum. This approach is used to describe the statistics of the breakup length for turbulent liquid jets in which turbulence is viewed as a random perturbation to capillary instability. This model agrees reasonably well with the data of Phinney[1].

VI. ACKNOWLEDGEMENT

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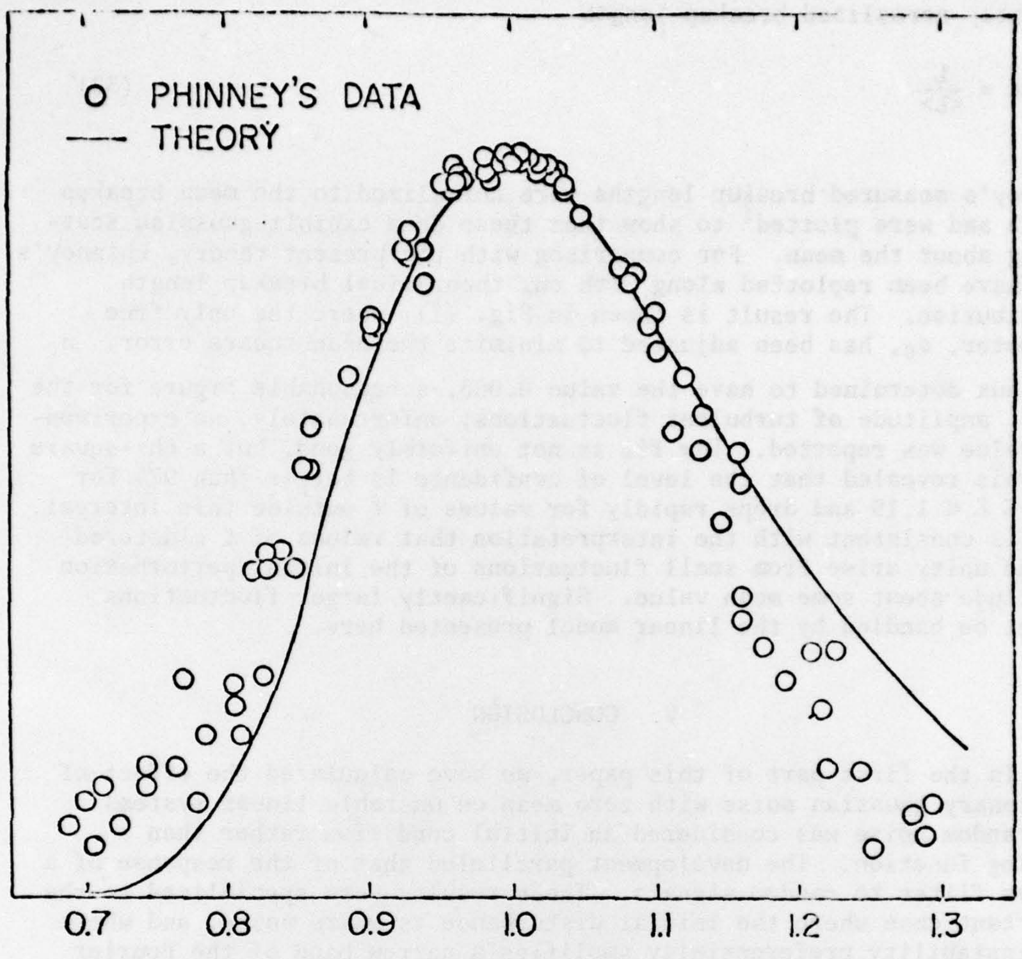

Figure 1. The statistical distribution of the normalized breakup length ℓ is given (in arbitrary units) as a function of ℓ.

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5. M. Schwartz, Information Transmission, Modulation and Noise, McGraw Hill, New York (1970).

LIST OF SYMBOLS

a	equilibrium radius of jet
$A(t)$	amplitude function, equation (19)
d_o	jet diameter; $d_o = 2a$
f	probability density function
$F(k,t)$	time-dependent Fourier transform of impulse response, Eq. (3)
$G(k,t)$	time-dependent power spectrum, Eq. (7)
i	imaginary unit; $i = \sqrt{-1}$
I_o, I_1	modified Bessel functions of the first kind
k	wave number
L	jet breakup length, Eq. (26)
m,n	indices; $m,n = 1, 2, 3, \ldots, N$
N	upper bound
N_w	Weber number; $N_w = \frac{\rho d_o u_o^2}{T}$
$R(y,t)$	autocorrelation function, Eq. (4)
t	time coordinate
U_o	jet velocity
x,y	(same) space coordinate
λ	wavelength
$\lambda_{m,n}$	elements of the covariance matrix, Eq. (11)
Λ	covariance matrix, Eq. (12)
$\xi(x,t)$	envelope of $\psi(x,t)$, Eq. (23)
ρ	fluid density
$\sigma^2(t)$	variance of stochastic process; $\sigma^2(t) = R(o,t)$

LIST OF SYMBOLS (continued)

Σ_s	spectral bandwidth, Eq. (20)
τ	small time scale
$\phi(x,t)$	phase of (x,t), Eq. (23)
$\psi(x,t)$	mechanical field variable
$\Psi(k)$	Fourier transform of (x,o), Eq. (1)
$\omega(k)$	complex frequency
Ω	constant defined in Eq. (30)

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